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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit provides students with an opportunity to discuss the need for scale drawings and the techniques used in their construction. Students are introduced to the idea that one unit can represent another kind of unit. These ideas are used to build some initial concepts of function and lead into rational numbers via ratios. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

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Math And Science Teaching project

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UNIT XVIII

MAPPING

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

SE 017 536

MATHEMATICS
FOR THE
ELEMENTARY SCHOOL

Unit XVIII

Mapping

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We are deeply indebted to the many teachers who
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CONTENTS

TEACHER COMMENTARY		1
Activity 1:	Getting a Bird's Eye View	5
Activity 2:	Building a Scale Model	7
STORY:	Mrs. Brown Buys a New Carpet	9
Worksheet 1		12
Worksheet 2		13
Worksheet 3		15
Activity 3:	New Methods of Measuring with Flips and Flops	17
Worksheet 4		20
Activity 4:	Geometric Representations	21
STORY:	Mrs. Brown gets Help from Squareville	24
Activity 5:	Scale Drawings on Squareville	25
Worksheet 5		26
Activity 6:	Table Tops in Squareville	27
Teachers' Commentary for Worksheets 6 and 7		29
Worksheet 6		30
Worksheet 7		32
Teachers' Commentary for Worksheet 8		34
Worksheet 8		36
Teachers' Commentary for Worksheets 9 and 10		38
Worksheet 9		39
Worksheet 10		40
Activity 7:	Scale Relationships	41
Activity 8:	Making an Enlargement Table	44
Activity 9:	The Notation for Multiplication	46
Activity 10:	Practice in Multiplication	49
Activity 11:	Commutativity	53
Activity 12:	The Missing Link	54
Children's copy of "Mrs. Brown Buys a New Carpet"		S1-S9

Teacher Background

This unit provides the student with an opportunity to discuss the need for scale drawings and the techniques used in their construction. Students are introduced to the idea that one unit can represent one of another kind of unit. For example, one inch could represent one foot, which we write as 1 inch$\leftrightarrow$1 foot. We could also write 1 inch$\leftrightarrow$1 mile, 1 foot$\leftrightarrow$1 block, 1 paperclip-length$\leftrightarrow$1 desk-length.

We can also use one unit to represent several of the same units. For example, 1 inch$\leftrightarrow$3 inches, 1 foot$\leftrightarrow$6 feet, 1 paperclip length$\leftrightarrow$6 paperclip lengths.

To many people the word "mapping" means drawing a plan of a city, state or other area. A floor plan or blueprint may come to mind. Mathematicians use the word "mapping" in a way which is closely related to its everyday meaning. In mathematics, any rule by which we associate each member of a set A with a member of another set B is called a <u>function</u>, and the association itself is a <u>mapping</u>. <u>One-to-one correspondences</u> are special cases of mappings.

This mathematical sense of mapping is not so different from the familiar sense of a map as a diagram or representation of objects. A floor plan or street map could be considered sets of lines related in a certain way; (each line and the geometrical relations among lines)$\leftrightarrow$(a member of the set of outlines and geometrical relations displayed by physical objects).

The mathematical meaning of "mapping" is especially useful when we begin to deal with sets of entities such as numbers, which may have properties and relations which are not so immediately obvious as the geometrical properties of a map-diagram and its territory.

Suppose we consider a scale relationship in which 1 unit represents 3 units (1 unit$\leftrightarrow$3 units). From our past experience we would have little

difficulty filling in this chart:

1 ⟷ 3	
1	3
2	6
3	
4	
7	21
9	

We immediately think of the "multiplying by 3" relationship in filling out the table.

We can think of the $1 \longleftrightarrow 3$ relationship as providing a rule by which every member of a set A is associated with a particular member of a set B. This rule determines a mapping in the mathematical sense; that is, it determines a set whose members are pairs of associated elements, one member of the pair from set A and one from set B. A function or rule determining a mapping is sometimes denoted by the letter "f". For any member x of the set A, the member of set B associated with it is called f(x), which is read "f of x" or function of x. Then the above table (after it has been filled out) can be re-written as:

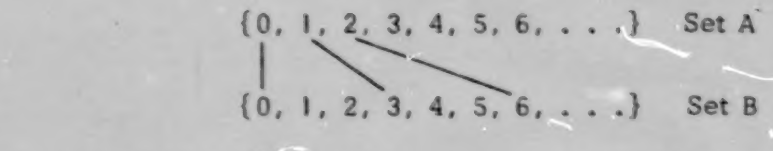
f(x) = 3x
f(1) = 3
f(2) = 6
f(3) = 9
f(4) = 12
f(7) = 21
f(9) = 27

Line 1 of this table tells us that when x, a member of set A, has the value 1, the member of set B - that is the f(x) - which is associated with x is 3. Line 2 tells us that the f(x) associated with x = 2 is 6, and so on.

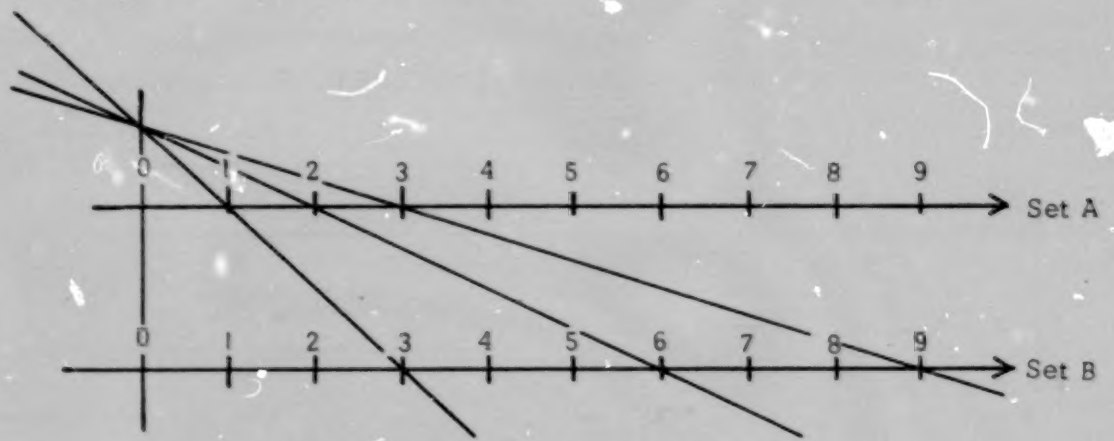
The function gives us the rule for picking out the f(x) which will be associated with any x we may select. We can describe this particular function more easily by saying f(x) = 3x. This formula tells us what member of set B is associated with a given member of set A. (In this case, set A and set B are the same, the set of all real numbers.) The rule given by this function is: with any x associate the value 3x in the mapping.

There are several ways by which we can illustrate the $1 \leftrightarrow 3$ relationship which exists between the members of set A and those of set B.

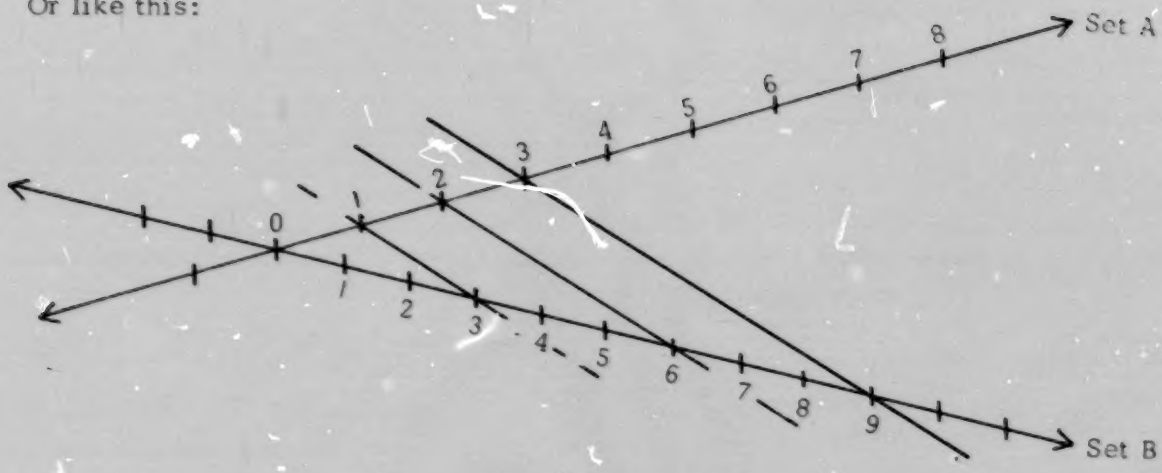
Some people prefer to write both sets of numbers and then use line segments to indicate that each member of set A is associated with a definite member of set B by the $1 \leftrightarrow 3$, i.e. f(x) = 3x, relationship.



Other people prefer to do it like this:



Or like this:



All of this will suggest that the notion of a mapping has many uses. Letting one unit represent another will first be used in scale drawing and changing scales. Our example in this section, where $1 \leftrightarrow 3$, also suggests that this idea is useful in multiplication.

Near the end of this unit, there is a discussion of the notation that is to be used in multiplication. (For example, $3 \cdot 2 = 6$.) The students are given cards with pairs of random numerals. These pairs of numerals can be used in multiplication exercises to provide the child with an opportunity for practice.

Activity 1 Getting a Bird's Eye View

Materials: Large sheets of unlined paper, a straight edge, pencils

The amount of time required to teach this activity is left to the discretion of the teacher. If the first idea is quickly grasped by the class, proceed to the next idea in the activity. It is not expected that all of the activity will be covered in one teaching period.

The teacher should begin this activity by drawing the shape of her classroom floor on the chalkboard. Then she should ask the class how she could represent the furniture in the classroom on this floor plan. She might begin by asking:

1. What would the teacher's desk look like on this plan?
2. Where will it be located?
3. What will the student's desk look like on this floor plan?
4. What shape will the table be on this floor plan?

Perhaps a child might draw a two dimensional front view of the desk on his floor plan showing the desk in perspective rather than the desk top. If so, the teacher should help the class visualize the room as if they were looking at it from the ceiling. Would the legs on the desks be seen?

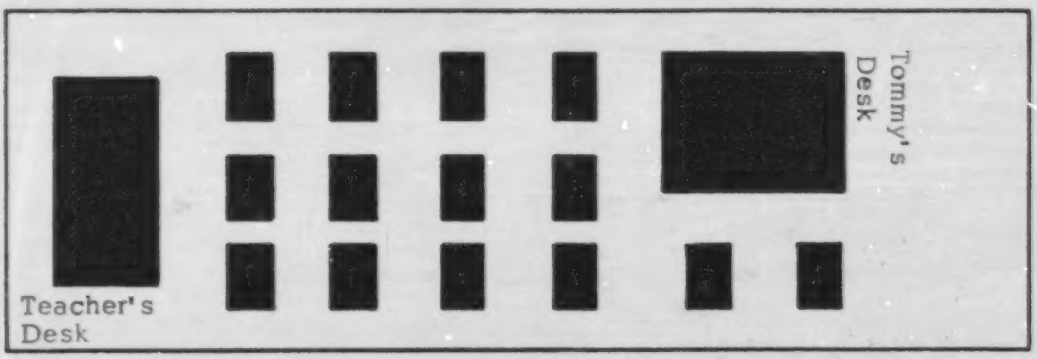
After eliciting a bird's eye view of the shapes of the furniture, draw these shapes on the board as examples.

When the students have determined the basic shapes of each piece of furniture in the room and the number of pieces, ask them to draw a floor plan of the classroom. Have them draw the different objects in their positions in the room. The student should be left free to draw this floor plan in any way he thinks is correct.

Then the children should meet in pairs or small groups to discuss and compare their drawngs. Encourage them to ask such questions as:

1. What is the number of objects in the room?
2. What is the size of each object?
3. What is the shape of each object?
4. What is the position of each object in the room?

While the children are discussing differences among their drawings, ask them if they drew some objects which appear out of proportion to the rest of the picture. Further explanation of the words "out of proportion" could be given in terms of "in comparison", "in relation to", and by examples. (Note Tommy's desk in the sample drawing. Is it in proportion to the other students' desks?)



Ask the class:

1. If you wanted to buy a cover for your desk, how could you be sure you had bought a piece of material of the right size and shape?
2. Suppose you wanted to buy a rug that covers the whole floor. Would you carry a full-sized plan of the floor with you to the store?

After this discussion, the students should agree that a map or plan must be smaller than the real object to be of any use when the real object is very large.

At this point it is suggested that the teacher set up a display table of maps, catalogues, pictures, house plans, doll house furniture and other

objects made to scale size. The children should be encouraged to explore the objects on this table. Perhaps the children can look for such articles at home and bring them to school to place on this table. After the table has been set up and explored, proceed to the second activity.

Activity 2 Building a Scale Model

Materials: Large sheets of unlined paper, pencil, straight edge

The goal of this activity is to have the student conclude that a scale model can be made by letting one inch on the scale drawing represent one foot on the full-sized object.

Discuss how a mail order company shows us the objects it sells. After someone says, "They send out catalogues which contain pictures of the real articles," ask:

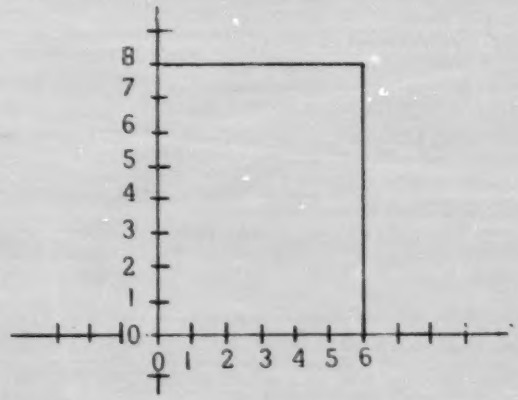
1. Are these pictures the exact size and shape of the real article?
2. How do they make these pictures so that each picture is the exact shape, but smaller in size? (They probably use a camera. Maybe an artist draws the pictures smaller.)
3. Imagine that you are told to draw a small picture of your desk top. What things do you notice about its shape? (There are four right angles, opposite sides are equal and some sides are longer than others.)

After this discussion, the teacher should draw a line on the chalkboard marked off in 1 inch units. Ask the class to pick out some object in the classroom (preferably rectangular) that is quite large, such as a table top or door. Have someone in the class measure the object with a 1 foot ruler. After the object has been measured and its length decided upon, the teacher should ask the students how they can use this information in making a scale drawing of the object.

The teacher should guide the students to see for themselves that one of the scale units on the chalkboard may stand for one of the foot units of the measured object. Therefore, if the object is 6 feet long, it will be represented by 6 inches on the scale drawing.

> This would be a good time to introduce the new notation
>
> 1 inch $\longleftrightarrow$ 1 foot
>
> Put this on the board and explain that when we make a scale drawing we will write this in the key to show that one inch on the scale drawing represents (stands for or corresponds to) one foot on the real object.

After the length has been measured and reduced to scale, repeat the above procedure for the width of the object. Possibly the following drawing will appear on the chalkboard.



This is an excellent opportunity for the students to compare the scale drawing with the actual object. The drawing will appear quite small on the chalkboard.

Next have each child choose a rectangularly shaped surface and ask him to construct a scale drawing of it. He can let 1 inch on his paper $\longleftrightarrow$ 1 foot on the real object. If possible, have the children measure objects which measure exactly in feet. Otherwise, they could approximate to the nearest foot. (6' 4 1/2" could be 6' and 6' 9 1/2" could be 7'.)

MRS. BROWN BUYS A NEW CARPET

Mrs. Brown needed to buy a new carpet for her living room. She took her yardstick and measured the old carpet. Here are its measurements:

length 16 feet
width 8 feet

She took a piece of paper that was 8 1/2 inches wide and 11 inches long. "I shall make a scale drawing of this carpet," she said. "I'll let 1 inch on the scale drawing stand for 1 foot on my real carpet."

You can imagine how excited she was as she began her drawing!

> Pass out paper that is 8 1/2 inches wide and 11 inches long, and ask the students to make a scale drawing of the carpet as Mrs. Brown did. After they have had an opportunity to try making the drawing, discuss the following questions:
> 1. How wide is the carpet? (8 feet)
> How wide would the scale drawing be? (8 inches)
> How wide is the paper? (8 1/2 inches)
> 2. How long is the carpet? (16 feet)
> How long would the scale drawing be? (16 inches)
> How long is the paper? (11 inches)
>
> Discuss with the students the problem they will have in making a scale drawing of the carpet.

To make a long story short, she looked but couldn't find a large enough piece of paper on which to make her 1 inch↔1 foot scale drawing. So, she rolled up her carpet, put it under her arm, heavy as it was, and headed for the store.

> Ask the students if they think Mrs. Brown acted wisely. Lead them to a discussion of the two ideas which follow.

1. Some children may suggest letting 1 scale unit represent more than 1 real life unit. This is, of course, also correct. We will discuss this idea later on.
2. Try to bring out the idea of letting 1 scale unit (smaller than 1 inch) represent 1 foot.

As she went out the front door she met Tommy coming in. As you can imagine, a young boy is no match for a grown woman carrying a big carpet. BUMP, Tommy fell flat on the floor!

"This is very upsetting!" Tommy said as he picked himself up. "You shouldn't be carrying a big carpet like that! Just what do you intend to do with it?"

"You know perfectly well what I'm planning to do," glared Tommy's Mother. "All of these pop and coffee spots are going to be replaced by a wonderfully soft, new carpet. Since you are personally responsible for some of these spots, it is your duty to help me carry this carpet to the store. So, grab ahold and let's go."

"Oh, no!" Tommy replied quickly. "This is too heavy to carry that far. Why don't you just make a scale drawing and take 'the scale drawing' to the store?"

"That, my fine young friend, has already been tried," was the answer he received. "I just don't have a large enough piece of paper on which to make the scale unit smaller."

"Here I'll show you," said Tommy. He picked up his notebook and took out a piece of paper. On it he made two marks about an inch apart.

He said, "It is about 1 inch from A to B. When you used this for your scale unit, the scale drawing did not fit on the paper. Try putting A and B closer together, like this:

Use this distance from A to B as your scale unit and see what happens."

Mrs. Brown used this new scale unit to make her new scale drawing. She was very surprised to find that her new scale drawing would fit on her sheet of paper. She smiled at Tommy and said, "Thank you, son. You'll be glad to know that you've just saved yourself a trip to the store."

Worksheet 1 Name_____

1 inch⟷1 foot

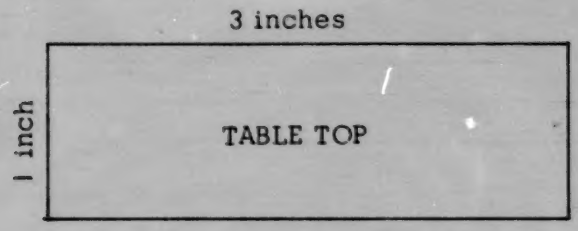
1. 1 inch⟷1 foot means that we will let one _____ on the scale drawing represent one _____ on the large object.

2. If a table is 6 feet long, then the scale drawing will be _____ long.

3. If the table is 3 feet wide, then the scale drawing will be _____ wide.

4. Measure the length of the scale drawing below in inches. How many feet long would the real object be? _____

5. Here is a scale drawing. Suppose the real table top is 3 feet long. Circle two things in the scale drawing that show this.



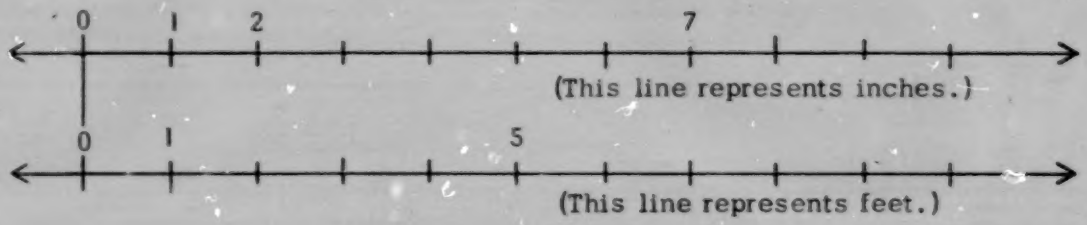
Key: 1 inch⟷1 foot

1. inch, foot
2. 6 inches
3. 3 inches
4. 5 feet long
5. 3 inches

Key: 1 inch⟷1 foot

Worksheet 2 Name_____

1. Below are two number lines where 1 inch ⟷ 1 foot. Place some more numerals in their correct places.



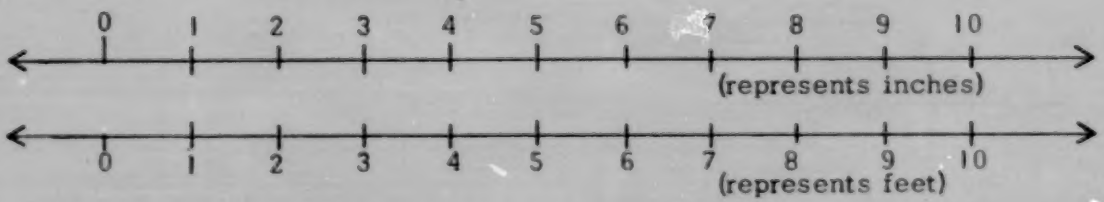
2. Fill in this chart.

Inches on scale drawing	0	1	2	3	4	5	6	7	8	9
Feet on real object										

3. Below are two lines.

The chart in exercise 2 shows that 0 inches on the scale drawing represent 0 feet on the real object.

Draw a line through the 0 points of both lines below, as shown in step 1.



 Name_____

4. From the chart we see that $\begin{bmatrix}1\\1\end{bmatrix}$ inches on the scale drawing represent feet on the real object.

Draw a line through these points.

5. Continue drawing lines through the points: $\begin{bmatrix}2\\2\end{bmatrix}$, $\begin{bmatrix}3\\3\end{bmatrix}$, $\begin{bmatrix}4\\4\end{bmatrix}$, etc.

What do you notice about the set of lines you have drawn?_____

_____

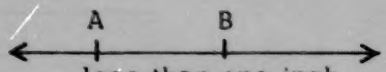
_____

Teacher's Copy

Worksheet 3 Name_____

Have each child choose some scale unit below that is smaller than 1 inch, as Mrs. Brown did.

one inch



less than one inch
new scale unit

Call the new scale unit a flip. Cut out the ruler and lay it along the flip, marking off a ruler of flip units. Have the children label the ruler "flip ruler."

Using your new scale of flips, see if you can make a scale drawing of the carpet on this sheet of paper. When you have finished, compare your scale drawing with some of your classmates' scale drawings.

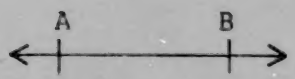
Some children might not make their units equal in marking off their ruler. Ask the children how they could make their scale units the same distance apart (A compass would be one way.)

In order to use this scale later, it would be helpful to give it some name: the children might call these units twinkles, tums, smidgens, or some other name. Because this will be used in other activities later we will call them 'flips'.

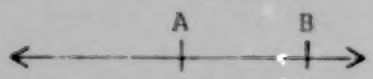
The students should come up with a variety of differently-sized drawings, since they will have chosen different scales. Point out that they all represent the same carpet, and that many different scales can be used in representing the same object.

Worksheet 3 Name_____

Choose some scale unit that is shorter than 1 inch, as Mrs. Brown did.



one inch



one of my
new scale units

Call the new scale unit a flip. Cut out the ruler and lay it along the flip, marking off a ruler of flip units. Label the ruler "flip ruler."

Using your new scale of flips, see if you can make a scale drawing of the carpet on this sheet of paper. When you have finished, compare your scale drawing with some of your classmates' scale drawings.

Activity 3 New Methods of Measuring with Flips and Flops

Materials: flip scale, pencil, straight edge, a school book (any size), and compass

The purpose of this activity is to introduce two arbitrary units of measure, rather than using inches and feet. In the previous activities, we first used one-inch units for the scale drawings and one-foot units for measuring the real objects. Then, in the story about Mrs. Brown, we developed the use of a scale unit smaller than one inch, while still using one-foot units to measure the real objects.

This activity involves measuring some object smaller than one foot. If students use 1 inch↔1 foot, they might have difficulty making their scale drawings. In other words, the dimensions of their scale drawing will be less than 1 inch.

This is an excellent opportunity for the children to discover that a unit smaller than one foot could be used to measure the object. By letting each child use his own scale unit (such as the flip) rather than an inch, and a new unit (less than one foot) to measure the real object, the child will have experience relating two arbitrary units of measure.

Review with the children the use they made of the ruler in Worksheet 3. Remind them they made a scale unit smaller than one inch when they made their flip scale. This represented one foot of the real object.

Now ask each student to measure a book and make a scale drawing of it, using their flip units. (1 flip↔1 foot)

The children may discover that this is difficult to do because the book size is less than one foot. Encourage suggestions as to how they could make a larger scale drawing.

The students may decide to let one "flip" represent a unit of measure that is less than one foot or perhaps they may think of other ideas that are useful. Then ask:

1. Could we measure books using units other than feet?
2. Could we make up a unit of measure smaller than one foot? What could we call it?

For example, could we use the length of a thumb or one of our fingers for this new unit? Other examples the children might decide upon are the palm of the hand, the width of a wrist, or the distance between two fingers. This length should be greater than an inch, but less than a foot. We will refer to this unit of measure as the "flop" unit.

Next, give each child a piece of paper at least as large as the book he is measuring.

On the edge of your piece of paper, mark a new unit of measure, such as a "flop" unit. Mark off a scale with this new "flop" unit, as was done when making the "flip" unit. A compass would help make the units the same distance apart.

It might be helpful to practice relating the two arbitrary units of measure at this point. Discuss some situations similar to the ones below:

1. If we measure our book to be 6 "flop" units in length, how long would we draw it using our "flip" ruler? (6 flips)
2. If we measure our book to be 3 "flops" wide, how wide would we draw it using our "flip" ruler? (3 flips)
3. If our book measures between 4 and 5 "flops" long, how long would our scale drawing be? (Between 4 and 5 flips long)

Each child should measure his book in "flop" units and record the measurements in both "flip" and "flop" units on Worksheet 4. The teacher should be sure that the number of "flop" units are the same as the number of "flip" (scale) units. Then each child should make a scale drawing of his book using the "flip" ruler.

Each student should be encouraged to measure other objects such as his notebook, his box of water paints, other-sized books, etc., and make scale drawings of them. After the children have had time to make their scale drawings, compare and discuss their results.

Worksheet 4 Name_____

Measure a book using your flop scale and record the results below:

Width of book: ____ flop units

Length of book: ____ flop units

To make a scale drawing of the book you will need to use your flip scale, where

1 flip unit ⟷ 1 flop unit

Complete this chart:

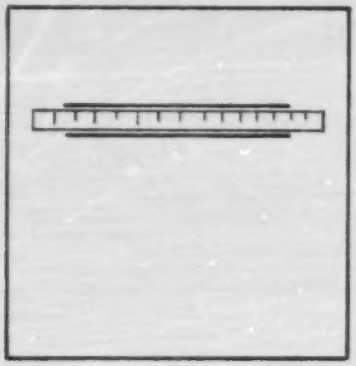
Width of book ⟷ Width of scale drawing	
_____flop units ⟷	_____flip units
Length of book ⟷ Length of scale drawing	
_____flop units ⟷	_____flip units

Make a scale drawing of the book using your flip scale units from the above chart.

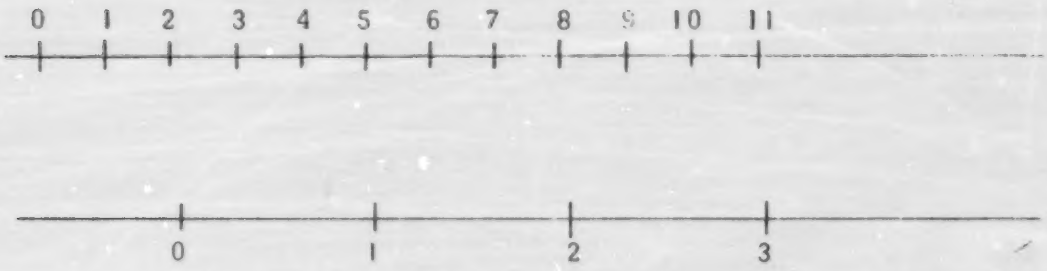
Activity 4 Geometric Representations

This activity is concerned with geometric representations for $I \leftrightarrow I$ relationships.

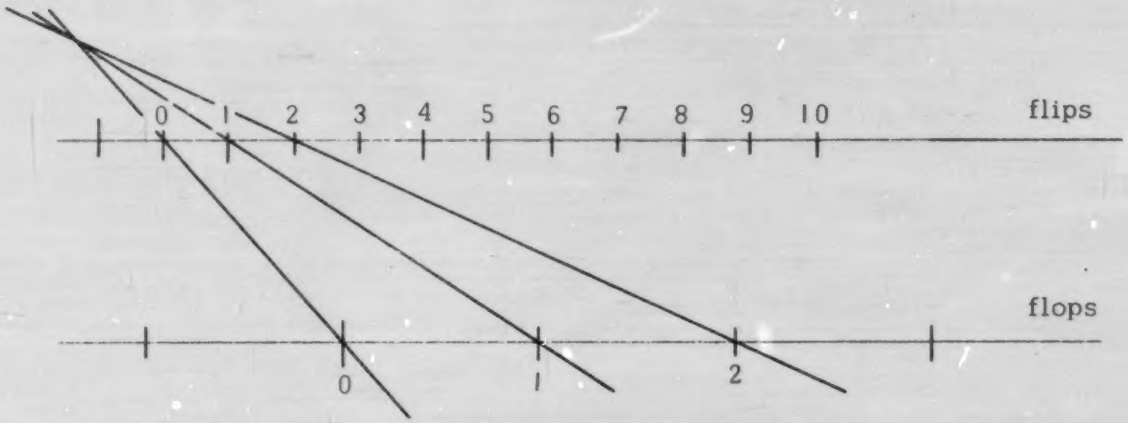
Have each student lay his ruler on top of the sheet and draw a line along each of the edges of the ruler. He should have two parallel line segments remaining on his paper.



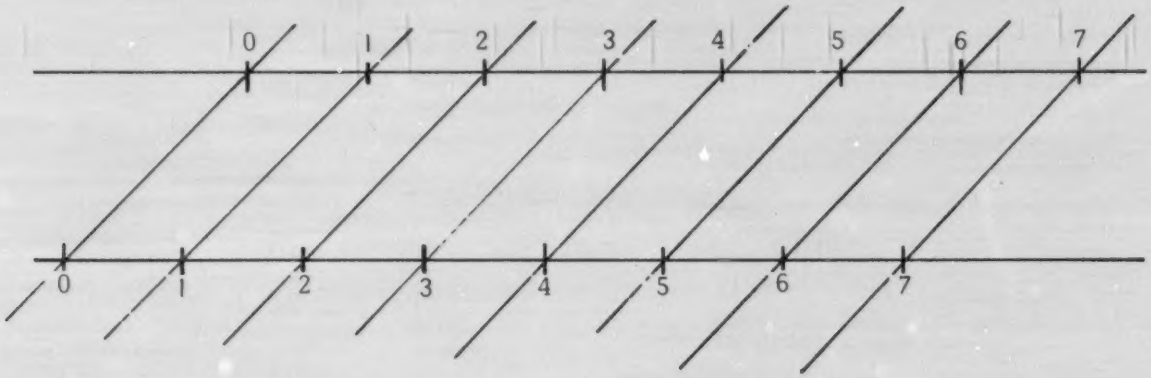
Then he can pick a point on each line and lay off equal segments on each line, laying off "flips" on one scale and "flops" on the other scale. One point on each line can be labeled 0, and all points to the right can be labeled in increasing numerical order. The 0 points need not be directly above each other.



If one line is labeled as the scale unit line (flips) and the other as the real unit line (flops), the 1 scale unit $\leftrightarrow$ 1 real unit relationship can be demonstrated by drawing line segments in the following manner:

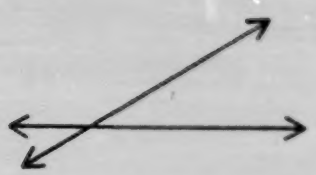


What do you notice about these lines if you extend them far enough? They all intersect at the same point. Then use the same unit for both lines. What do you notice?

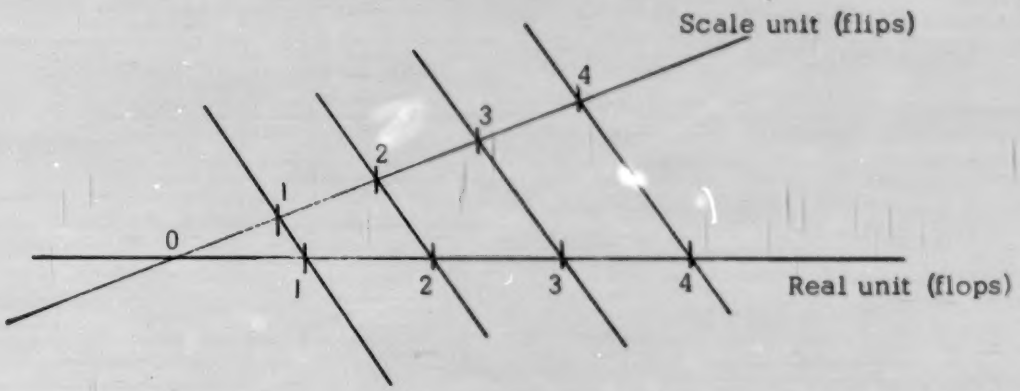


They are parallel.

Let us look at another geometric representation for mapping the 1$\leftrightarrow$1 relationship. We can have the children take a blank sheet of paper and draw two intersecting lines on that paper.



If they let the intersection of the two lines be the 0 point, units of lengths can be marked off on both lines. One line can be labeled as the "scale unit" line and the other as the "real unit" line.



Give the students the chance to explore this idea further. The children could use units of different length on each line. Do all of your connecting lines have a common property? All the connecting lines for a given relationship are parallel in the above construction.

MRS. BROWN GETS HELP FROM SQUAREVILLE

All is not well at Mrs. Brown's house. Tommy's idea of making a scale drawing of the carpet and taking the "scale drawing" to the store was a very good idea, but Mrs. Brown had trouble making her lines straight in the drawing.

"Oh, my!" said the man at the store. "I'm afraid your drawing is very confusing. The lines are crooked and crowded together. I can't tell much about the shape of your carpet."

Of course, Mrs. Brown was disappointed. At home, she talked the problem over with Tommy. He sat down and just thought about it. If he did not find a way of making a better scale drawing, he knew he would be making a trip to the store carrying a very heavy carpet!

Activity 5 Scale Drawings on Squareville

Materials: 8 1/2" x 11" graph paper, pencil, straight edge

Read the story, "Mrs. Brown gets help from Squareville." Discuss the following ideas with the children:

1. Would it be possible to draw rectangles on the map of Squareville?

Point out that it would be easy to draw rectangles on the map of Squareville because it is already marked with horizontal and vertical lines, which we can use for the sides and ends of objects. Thus we could use Squareville to make a scale drawing of a table top, or anything else we wanted to draw.

2. Have we ever made a scale drawing on the map of Squareville before?

The children will probably say no. At this point, indicate to them that by putting the Mayor's house, City Hall, and all the highways on the map of Squareville, we are making a scale drawing of a town.

The scale we used was "the side of 1 square on the Squareville map" $\longleftrightarrow$ "the length of 1 block in the real city." All maps are scale drawings. If there are any maps in the room, point out that these maps are also scale drawings. It is best to iqnore the globe, which presents different types of problems.

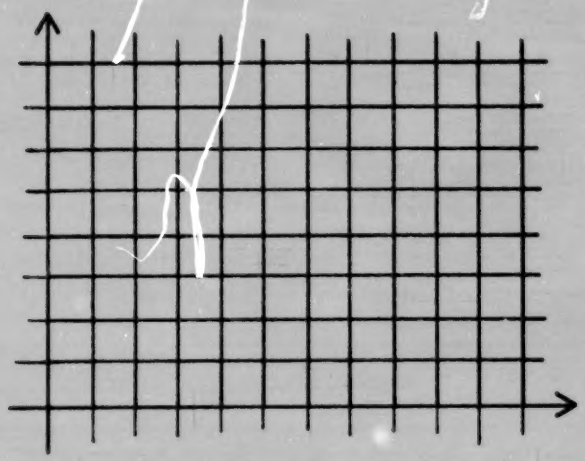
Could we make a map of Mrs. Brown's rug on Squareville? Do you think a map of Squareville could help Tommy and his mother?

At this point, instruct the children to make a map for Tommy. The children should be encouraged to write on the drawing what one unit represents. In other words, they should write out a key. For example, "1 unit on the Squareville map" $\longleftrightarrow$ "1 foot of the rug." Be sure that their Squareville map is at least 16 squares by 8 squares.

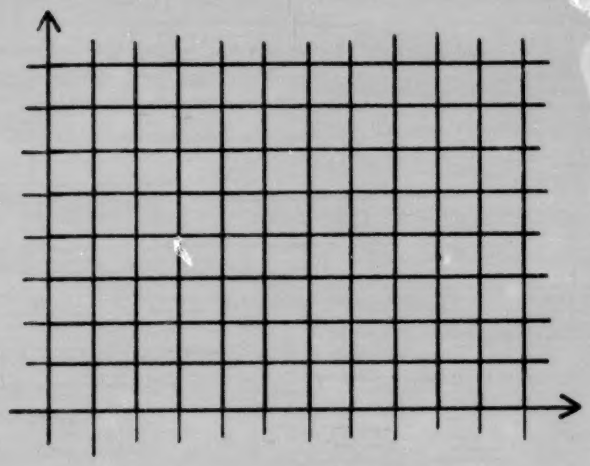
Worksheet 5 Name_____

Directions: Construct scale drawings on the graphs below where (1 unit on the graph)$\longleftrightarrow$(1 unit (feet) of the real object).

Table top - length, 4 feet; width, 2 feet



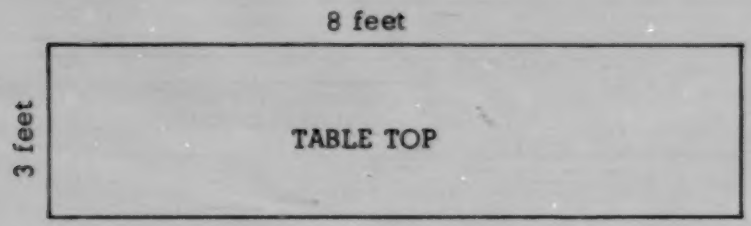
Sandbox - length, 7 feet; width, 5 feet



COMPARE YOUR DRAWING WITH ONE OF YOUR CLASSMATES.

Activity 6 Table Tops in Squareville

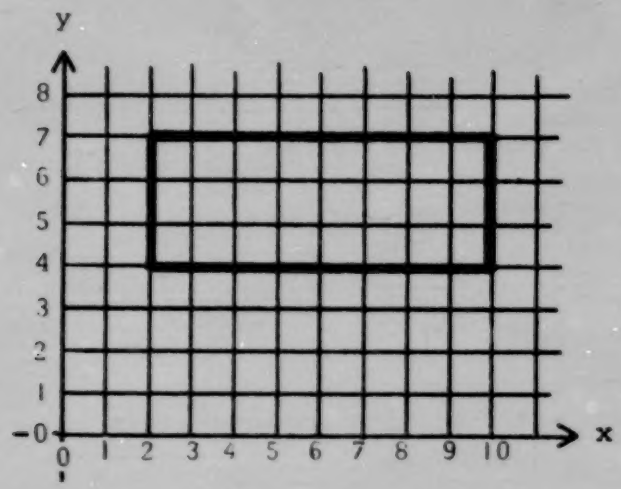
On the chalkboard draw a picture of some object whose shape is rectangular or square, such as a table top. Write its length and width on the drawing.



Give each child a map of Squareville and ask each child to make a scale drawing of the table top. They are to make their scale drawing on the map of Squareville. Remind them to write a key on the map, such as:

Key: 1 unit of the Squareville map $\longleftrightarrow$ 1 foot of the real table

The drawing may look like this:



The table top could be placed a great many possible ways in Squareville. How many? Explore this idea by having each child write his corner addresses onto this map of Squareville.

Then construct the following chart on the chalkboard and let several children fill in the information for their table.

Location of Corners

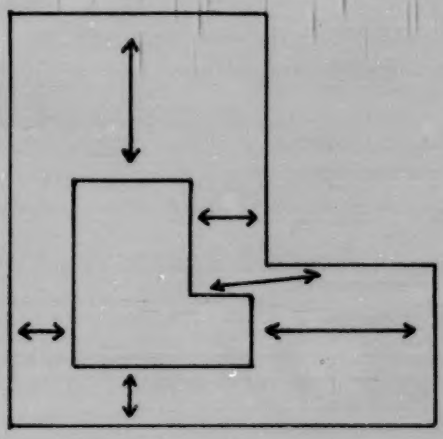
Name	Lower Left	Lower Right	Upper Right	Upper Left
John	(2,4)	(10,4)	(10,7)	(2,7)

Teachers' Commentary for Worksheets 6 and 7

In these worksheets the children are to enlarge drawings by using two different sizes of graph paper. One small square on the finely ruled paper represents one square on the paper with larger divisions. The children learn this by using Worksheet 6.

Explain how Worksheet 6 is done by taking the drawing at the top left of the worksheet and enlarging it to produce the drawing at the top right. (The key might give the children a hint as to how this was done.) Every line in the small drawing represents a longer line drawn on the large drawing.

To make sure the children understand this, the teacher should point out a line on the small drawing and ask which line on the large drawing it represents. It may be helpful to put the sample enlargement on the chalkboard and mark the corresponding lines in each drawing with arrowed lines. Using different colors of chalk will retain the simplicity of the drawing.

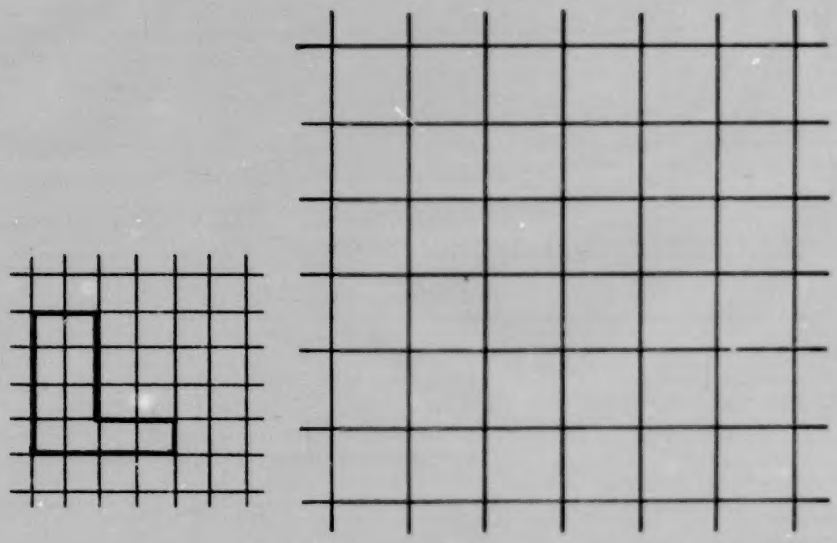


As children point out corresponding lines in the two figures, draw in the arrows.

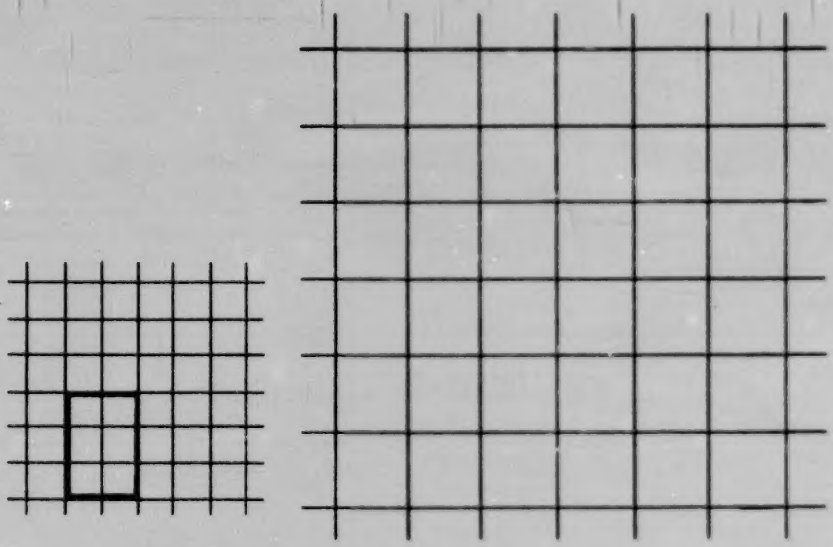
Worksheet 6

Name_____

Directions: Enlarge the small drawing on the large graph paper.



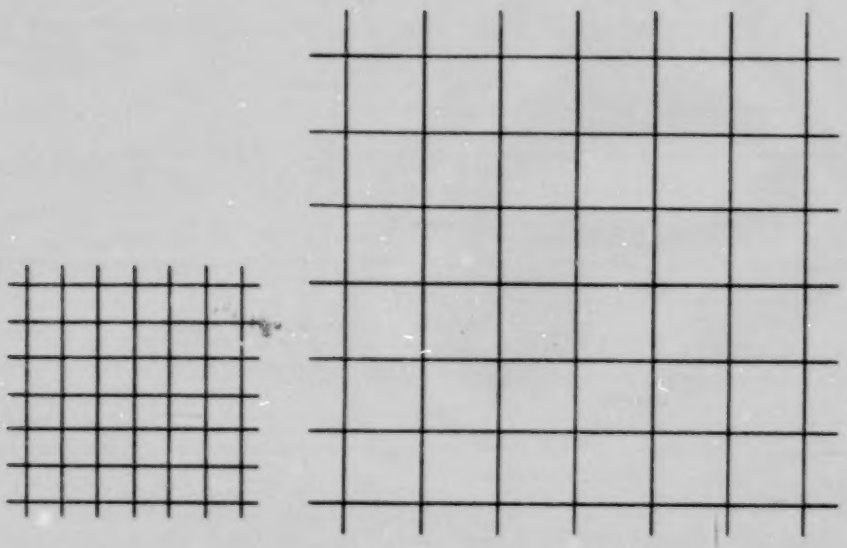
Key: 1 small unit ⟷ 1 large unit



Key: 1 small unit ⟷ 1 large unit

Name_____

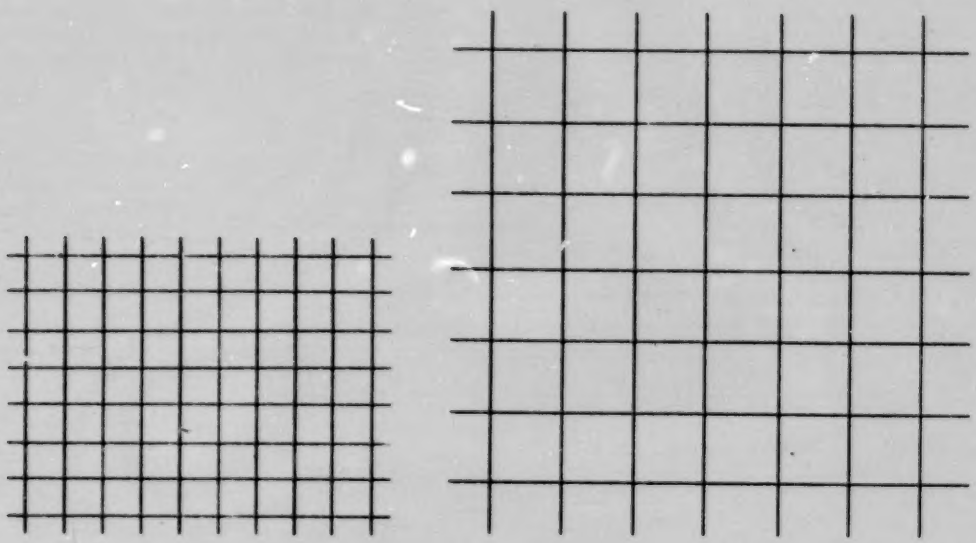
Directions: Make your own small drawing and enlarge it cn the large graph paper.



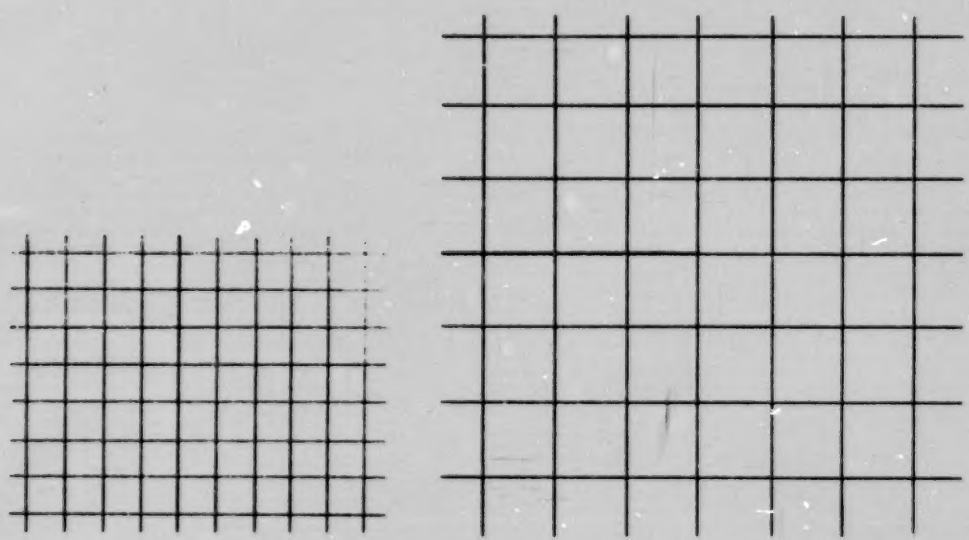
Key: 1 small unit ⟷ 1 large unit

Worksheet 7 Name_____

Directions: Make a drawing on the small graph. Have another student enlarge it on the large graph.

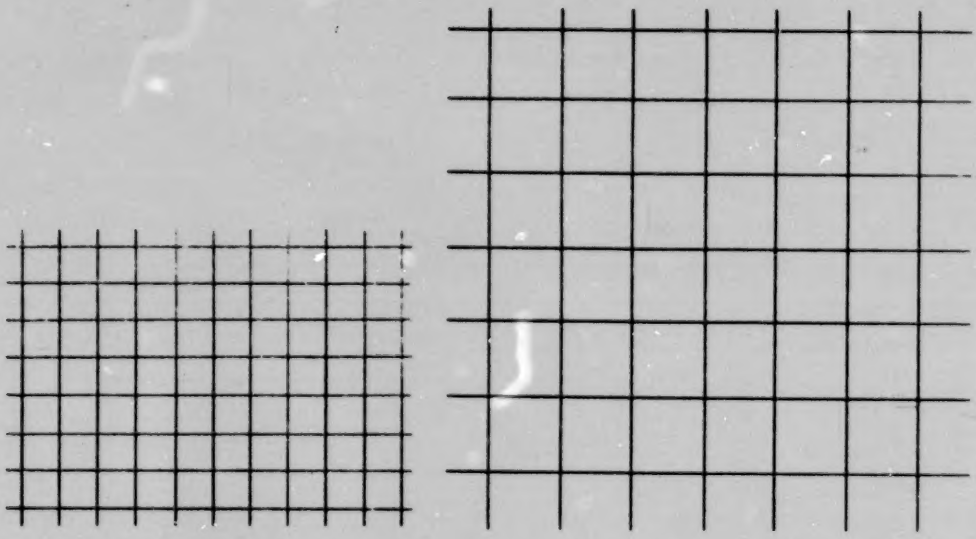


Key: 1 small unit ⟷ 1 large unit



Key: 1 small unit ⟷ 1 large unit

Name_____

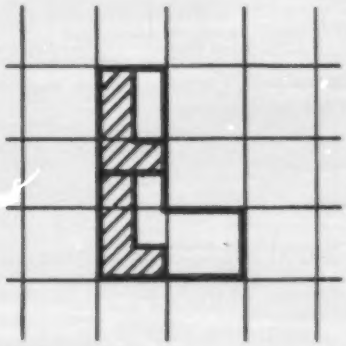


Key: 1 small unit ⟷ 1 large unit

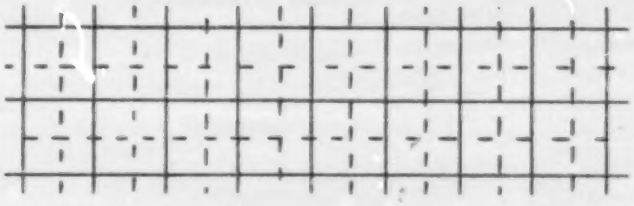
Teachers' Commentary for Worksheet 8

Have the children learn to enlarge drawings on Squareville using the smaller graph size. They are to be led to the idea of making a multiplication-by-two table.

Cut out the smaller drawing from Worksheet 8 and place it on top of its enlargement. Ask the children how many smaller drawings laid end-to-end would be needed to stretch across the bottom of the larger drawing. Two are needed. Compare the heights of the two drawings. How many of the small drawings would fit "head-to-toe" in the long part of the large drawing?



Ask how many small units (lengths of the side of a square on the small graph) it takes to make up one large unit (the length of the side of a square on the large graph). Do not ask how many small squares make up one large square, since this leads into the notion of multiplication which comes later in this unit. We are interested here only in lengths and not area. All comparisons of the two drawings must be made in terms of lengths of various lines. Discuss what would happen if lines were drawn both vertically and horizontally between the graph lines of the large map of Squareville, as in the following figure:



In Exercises 2 and 3 of Worksheet 8 we divided each large unit into 2 equal parts and found each of the large units to be 2 small units long.

Suggest to the children that they might do the same on the large drawing in Exercise 1 of Worksheet 8. (This would take considerable time and they might not arrange the lines exactly in the middle of the squares.) They might cut out the drawing, line it up with the lines on the graph on the right of Exercise 2, and trace it.

Discuss with the children what they should put in the key for this drawing. Note to the children that large units are not used on this graph paper. Find a line on the small drawing which is one unit long and ask the children how many small units there are in the line it represents on the large diagram.

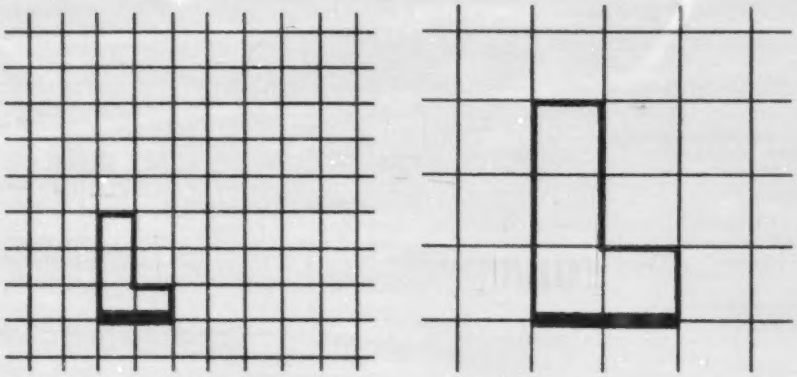
Key: 1 small unit on the left drawing ↔ 2 small units on the right drawing

Have the children compare the two keys that are now on the worksheet. The key for Exercise 1 says 1 small unit ↔ 1 large unit, while for Exercise 2 says 1 small unit ↔ 2 large units. Since the drawings are exactly the same, the keys lead us to conclude that two small units can represent one large one. In the two drawings at the bottom of Worksheet 8, the children should find the line on the large diagram which corresponds to the heavy line on the small diagram, and write the lengths of both lines in the spaces provided.

Worksheet 8 Name_____

Directions: Cut out the small drawing. Place it on the large drawing. Compare the heavy lines of the two figures. How many small heavy lines fit across the large heavy line?

Exercise 1

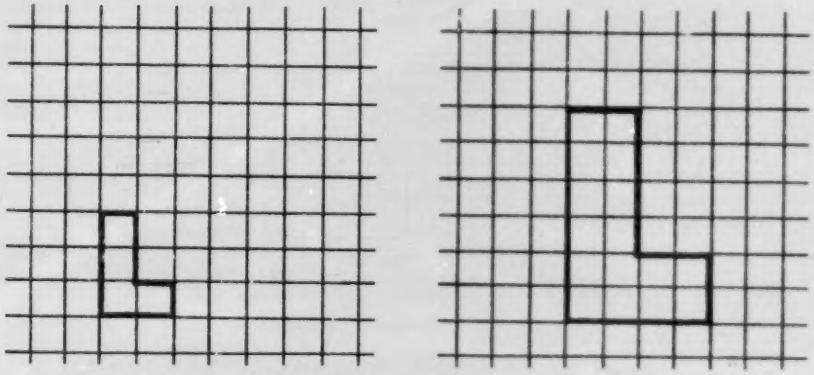


Key: 1 small unit ⟷ 1 large unit

Name_____

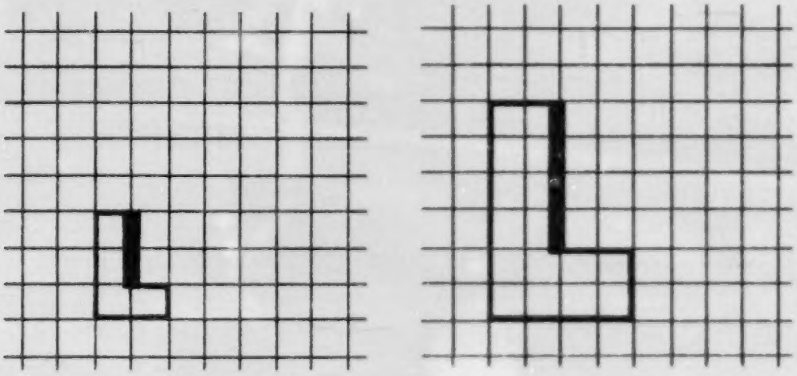
Directions: Compare the two scale drawings below. They are much like the ones on the other page, but there is a major difference. What is it? Can you fill in the key below? (Clue: How many small units on the drawing in the lower right are represented by one small unit on the drawing in the lower left?)

Exercise 2



Key: 1 small unit ⟷ _____

Exercise 3



3. How long is the heavy line on the small drawing? _____small units long.

How long is the heavy line on the large drawing? _____small units long.

Teachers' Commentary for Worksheets 9 and 10

At the top of Worksheet 9, there is a pair of graphs, both with grids of small squares to guide them. If some children have difficulty, suggest they draw a two-unit line on the large drawing for each of the one-unit lines on the small drawing. Remind them that the key 1$\longleftrightarrow$2 units gives them directions for making enlargements.

The second pair of graphs should be treated similarly. The small drawing on the left should be enlarged at the right. The children can find a line of the length they want on the small drawing, then determine how long the corresponding line is on the large drawing by counting the number of small units in it.

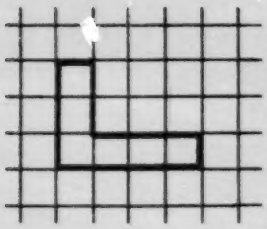
Ask the children how we might avoid having to make and enlarge a new drawing each time we wanted to enlarge a line. We might make a complete enlarged drawing once for each initial figure whose sides measure a certain number of units and list the results. Whenever we have a line to enlarge, we can count the number of units in it, look it up in our list, and know right away how long the enlarged line should be.

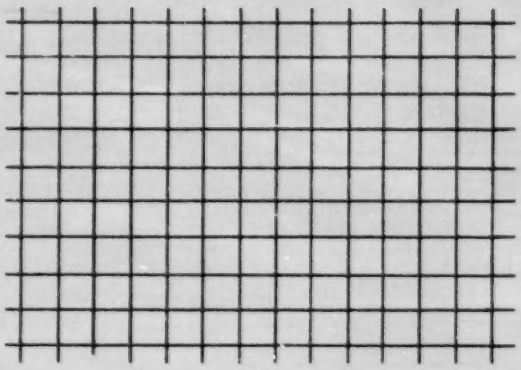
Fill in the appropriate numbers after the children have completed Worksheet 9, then have them do Worksheet 10.

Length of a line in the small drawing (Number of units long)				
Length of a line in the large drawing (Number of units long)				

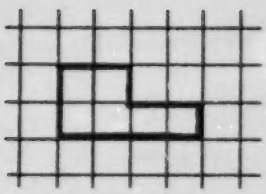
Worksheet 9 Name_____

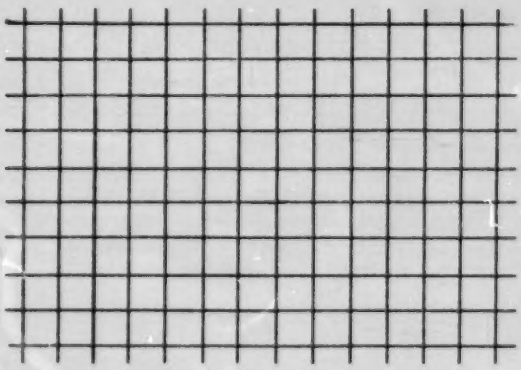
Directions: Enlarge the small drawing on the map of Squareville at the right.





Key: 1 small unit ⟷ 2 small units
on the left on the right

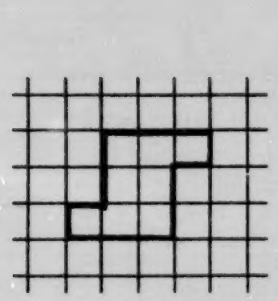


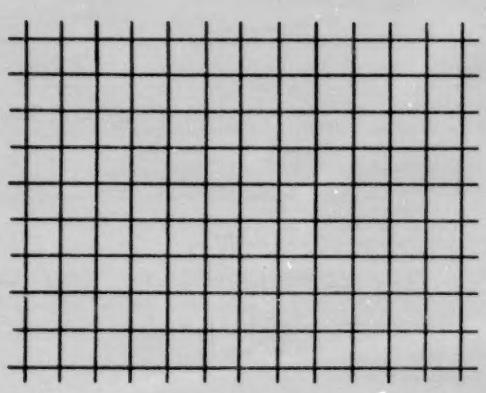


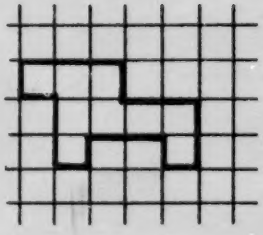
Key: 1 small unit ⟷ 2 small units
on the left on the right

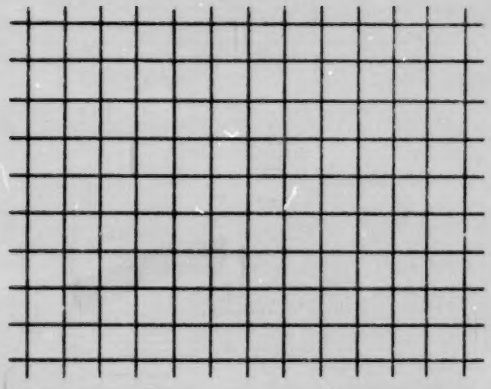
Worksheet 10 Name_____

Can you enlarge these drawings? Use any key that you wish, just so the new drawing is larger than the old. If you wish, get a larger grid from your teacher.









Activity 7 Scale Relationships

Have each child work with these relationships until they become familiar with them. It is not necessary to use only this notation. Encourage the children to think of others.

$1 \longleftrightarrow 4$ $\qquad$ $1 \longleftrightarrow 6$

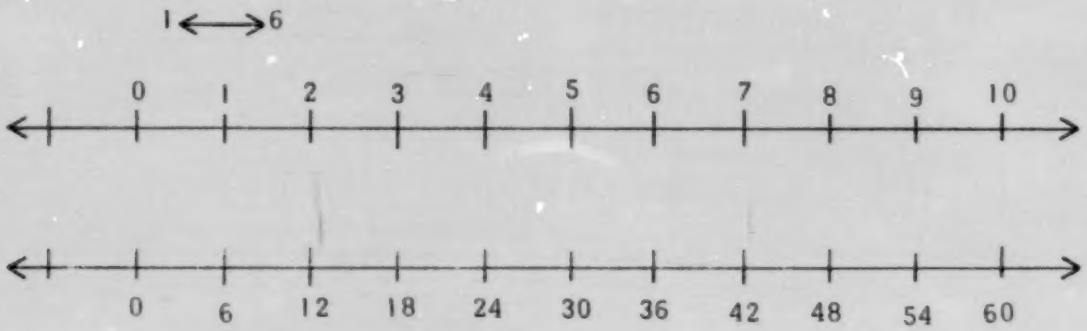
$1 \longleftrightarrow 5$ $\qquad$ $1 \longleftrightarrow 3$

$1 \longleftrightarrow 9$ $\qquad$ $1 \longleftrightarrow 0$

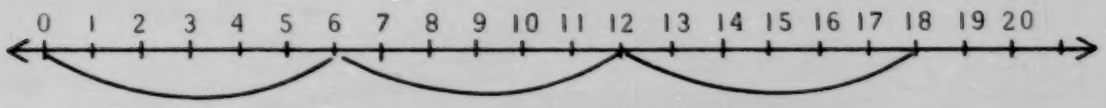
After the $1 \longleftrightarrow 1$ and $1 \longleftrightarrow 2$ relationships have been explored, form the children in groups and have them pick one of the relationships above for further study.

Suppose one group of 3 friends decides to explore the $1 \longleftrightarrow 6$ relationship. They could choose one of the following methods:

1. Draw two number lines with two scales and fill in the correct numerals.



2. They may need to discover the numbers by taking jumps of 6 on a number line.



3. Mapping of sets:

1 ⟷ 6

Set A = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...}

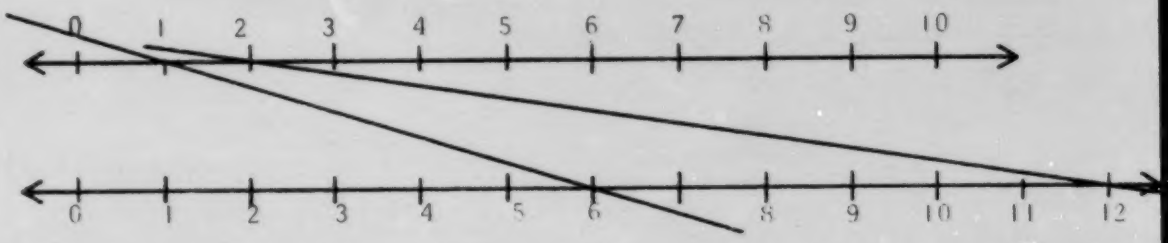
Set B = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ...}

4. Make a chart:

1 ⟷ 6

Scale	0	1	2	3			n
Real	0	6	12	18			6n

5. Show the correspondence on parallel number lines.



6. Prepare a set of two questions to be answered by each student in the class.

   Use the 1 $\longleftrightarrow$ 6 relationship. Plug in some numbers in one blank and try to decide the number that goes in the other blank.

> Example 1
>
> John has 6 marbles in each pocket. He has_____pockets.
> If you counted all the marbles in all his pockets, you would find that he had____marbles in his pockets.

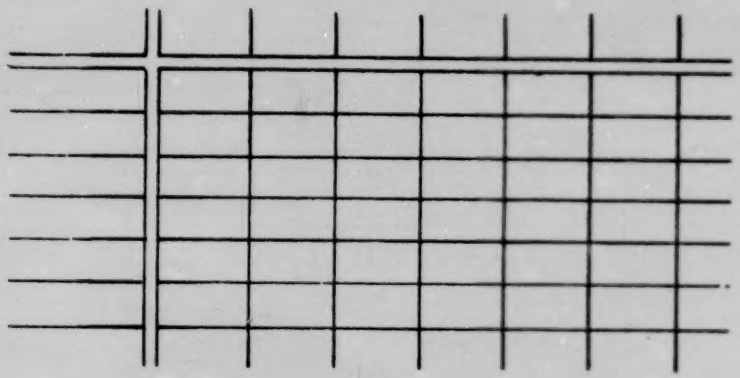
or

> Example 2
>
> There are 6 chairs in each row. There are_____rows so there are_____chairs in all.

Activity 8 Making an Enlargement Table

We could continue making drawings and tables to show these scale relationships, but it is more convenient to put all our tables together into one chart. We will call this our "Enlarging Chart".

The blank form for this chart is shown below:



Put this chart on the chalkboard and discuss how we might list all of our tables on it. For example:

	0	1	2	3	4	5	6				
1↔2	0	2	4	6	8	10	12				
1↔7											
1↔1											

The children should be able to build their own "Enlarging Chart". They should include the 1↔3 row (taken from the 1↔3 table), the 1↔4 row, and rows from any other table they have constructed using their number

lines. They will then have a table that is similar to this:

	0	1	2	3	4	5	6
1↔2	0	2	4	6	8	10	12
1↔3	0	3	6	9	12	15	18
1↔4	0	4	8	12	16	20	24
1↔7	0	7	14	21	28	35	42
1↔1	0	1	2	3	4	5	6

The children should be able to build their own charts and put in some numbers. At this point in the instruction, proceed with care. Adults usually order the integers along the left side and top of the chart, like the chart above. It is perfectly all right to do this, but it is not necessary. Do not suggest or force the children to order the numbers at this time. The chart below is just as acceptable.

	2	1	7	4	3	5			
1↔5									
1↔2									
1↔9									
1↔4									
1↔0									

After the children have finished making their tables, they may abbreviate the 1↔2, 1↔3, 1↔5, etc., row headings. These can be replaced by 2, 3, 5, etc. Do not force this change; however, it should be encouraged if the children suggest it.

When the students have finished and checked their charts, they can look for interesting patterns. If the original chart was not arranged in numerical order, new patterns may be found by ordering the numerals along the top and left side of the table.

Activity 9 The Notation for Multiplication

1. Draw attention to the Enlarging Chart and have some child circle numerals that represent these three items:

   a. Column numeral (numeral at the top of a column)
   b. Row numeral (numeral at the left of a row)
   c. The number that is in the chosen row and column.

Number of units in a line on the small drawing

Number of units on the large drawing represented by one unit on the small drawing	0	1	2	3	4	5
1↔2	0	2	4	6	8	10
1↔3	0	3	6	9	12	15
1↔4	0	4	8	12	16	20

The numerals in the boxes give the length of the corresponding line on the large drawing.

2. Suppose a child chooses these three numerals: 3, 2, 6. Ask him to explain the meaning of each numeral. As he does, write the information

on the chalkboard in this manner:

3 units	2 units	6 units
Length on small drawing	Length on large drawing represented by one unit on small drawing	Length on large drawing

3. Choose 3 more numerals and explain, in writing, what is meant. Have each child write the explanation on his own paper.

4. Indicate that there is a shorthand method of recording this information. Some child will probably suggest writing:

$$3 \times 2 = 6$$

5. Tell the children some people would commonly write it with the sign "x". Ask if anyone knows another sign that is used by mathematicians. Explain that they use the sign of (·). This is how the sentence appears with this new sign:

$$3 \cdot 2 = 6$$

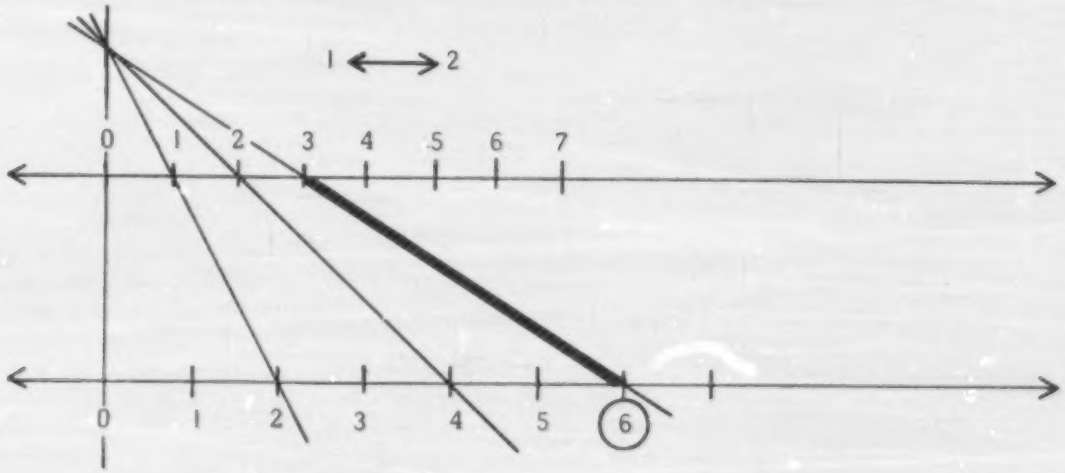
> Caution the children as to the placement of this multiplication sign. It must not appear as a period or a decimal point.

6. Ask if anyone knows how this sentence is read. It is read as "3 times 2 equals 6".

7. Choose other sets of numerals. Write and read them in the new way.

8. Draw a pair of number lines on the board, one under the other, and number them. Ask the children if they can show what $3 \cdot 2 = 6$ means on the number line. The upper one should be marked off in an arbitrary unit, then marks should be placed on the lower line under every third

mark on the upper line.



9. Choose some pairs of numerals. Multiply them on number lines.

Other Applications of Multiplication

The students will quickly discover that the mathematical sentence "3 · 2 = 6" can be applied to some situations other than scale drawings. These next activities can be used to reinforce the ideas of:

a. arrangement of objects in rows and columns

b. commutativity of multiplication.

Materials needed for each child:

a. Enlargement Chart

b. Random Number Cards A and B

c. Set of 100 items such as beads, squares of paper, and paper circles to construct arrays.

Activity 10 Practice in Multiplication

After each child has all the necessary materials, one might begin by writing these sentences on the chalkboard.

There are_____rows of chairs in the room. There are_____ chairs in each row. How many chairs are there altogether?

RANDOM NUMBER CARD A

1,1	1,6	4,3	6,3	1,8	7,5	1,6	1,3	7,6
2,1	2,1	5,9	1,7	9,1	7,6	8,3	1,5	8,6
1,0	4,3	8,4	4,4	8,2	6,6	5,5	8,3	7,3
3,6	7,9	2,2	6,2	3,6	3,3	2,6	6,6	6,5
7,3	9,4	4,0	4,7	7,6	1,2	0,3	2,5	2,4
4,9	5,6	3,1	2,8	7,2	1,4	0,6	3,9	3,1
6,4	2,0	8,4	8,2	3,7	4,1	7,0	1,7	3,1
5,1	4,8	6,7	2,8	7,5	3,8	6,1	5,2	9,3
9,9	7,5	6,2	6,3	6,0	6,4	5,1	6,1	7,9
7,1	3,2	5,5	5,2	1,7	1,3	0,2	5,7	2,9

RANDOM NUMBER CARD B								
6,5	2,8	5,9	7,1	9,8	1,2	1,3	8,5	3,0
1,7	2,6	4,5	7,3	2,7	3,8	2,2	4,2	9,3
9,5	6,3	9,9	9,7	5,4	3,1	1,9	9,9	2,5
6,1	5,5	5,7	6,4	0,4	0,5	6,8	9,8	7,1
7,8	1,3	7,9	8,7	6,8	8,4	9,8	7,2	8,7
6,2	4,9	0,9	9,2	1,5	0,4	6,8	9,8	7,1
2,4	2,1	6,6	3,4	4,4	2,1	2,8	3,0	7,0
1,6	9,7	5,9	5,4	2,8	3,3	2,2	6,5	5,9
5,9	1,3	8,3	9,5	4,2	7,1	1,6	8,5	7,6
2,9	4,7	8,5	9,6	5,2	5,0	4,1	4,3	1,9

Have each child choose one pair of numerals from one of the Random Number Cards. These numerals should then be filled into the blanks of the sentences and an answer provided.

The students will have chosen different pairs; therefore, there will be different results in general.

Many children will be eager to fill in the numerals and give their results. Allow them to do this.

As these numerals are being filled in, suggest that it might be a good idea to record them. If no one suggests writing them as multiplication sentences, indicate this method of recording.

Number of Rows	Number of Chairs in Each Row	Number of Chairs Altogether	Mathematical Sentence
3	2	6	$3 \cdot 2 = 6$
4	7	28	$4 \cdot 7 = 28$

RANDOM NUMBER CARD A

1,1	1,6	4,3	6,3	1,8	7,5	1,6	1,3	7,6
2,1	2,1	5,9	1,7	9,1	7,6	8,3	1,5	8,6
1,0	4,3	8,4	4,4	8,2	6,6	5,5	8,3	7,3
3,6	7,9	2,2	6,2	3,6	3,3	2,6	6,6	6,5
7,3	9,4	4,0	4,7	7,6	1,2	0,3	2,5	2,4
4,9	5,6	3,1	2,8	7,2	1,4	0,6	3,9	3,1
6,4	2,0	8,4	8,2	3,7	4,1	7,0	1,7	3,1
5,1	4,8	6,7	2,8	7,5	3,8	6,1	5,2	9,3
9,9	7,5	6,2	6,3	6,0	6,4	5,1	6,1	7,9
7,1	3,2	5,5	5,2	1,7	1,3	0,2	5,7	2,9

RANDOM NUMBER CARD B

6,5	2,8	5,9	7,1	9,8	1,2	1,3	8,5	3,0
1,7	2,6	4,5	7,3	2,7	3,8	2,2	4,2	9,3
9,5	6,3	9,9	9,7	5,4	3,1	1,9	9,9	2,5
6,1	5,5	5,7	6,4	0,4	0,5	6,8	9,8	7,1
7,8	1,3	7,9	8,7	6,8	8,4	9,8	7,2	8,7
6,2	4,9	0,9	9,2	1,5	0,4	6,8	9,8	7,1
2,4	2,1	6,6	3,4	4,4	2,1	2,8	3,0	7,0
1,6	9,7	5,9	5,4	2,8	3,3	2,2	6,5	5,9
5,9	1,3	8,3	9,5	4,2	7,1	1,6	8,5	7,6
2,9	4,7	8,5	9,6	5,2	5,0	4,1	4,3	1,9

Activity 11 Commutativity

Have the children fill in the blanks below with numerals below 10.

There are_____rows of chairs in the room. There are_____ chairs in each row. How many chairs are there altogether?

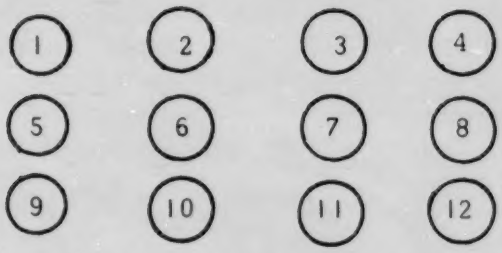
Ask each child to take some of his beads or paper circles, or whatever he has, and arrange them in the order stated in the sentence.

4 rows of chairs
3 chairs in a row
12 chairs altogether

$4 \cdot 3 = 12$

Suggest that some child reverse the order of the numerals between the number of rows and number of chairs in a row as shown in the blanks provided above.

Keeping the first arrangement of objects in front of you, take the objects and make a different picture to illustrate this new situation.



3 rows of chairs
4 chairs in each row
12 chairs in all

$3 \cdot 4 = 12$

Discuss the fact that the results are the same, but that the situation is different. Emphasize that the situation is different. (The commutative property of multiplication is at stake.)

Activity 12 The Missing Link

Put this table on the chalkboard. Have the children fill it out.

Number of rows	Number of chairs in each row	Number of chairs altogether	Mathematical Sentence
3	3	?	$3 \cdot 3 =$
2	4	?	$2 \cdot 4 =$
3	?	6	$3 \cdot \quad = 6$
4	?	16	$4 \cdot \quad = 16$
?	5	20	$\cdot 5 = 20$

Ask the children to solve problems of the following type:

$3 \cdot 3 = \square$

$7 \cdot \square = 14$

$\square \cdot 5 = 20$

Mrs. Brown Buys

A New Carpet

Illustrated by

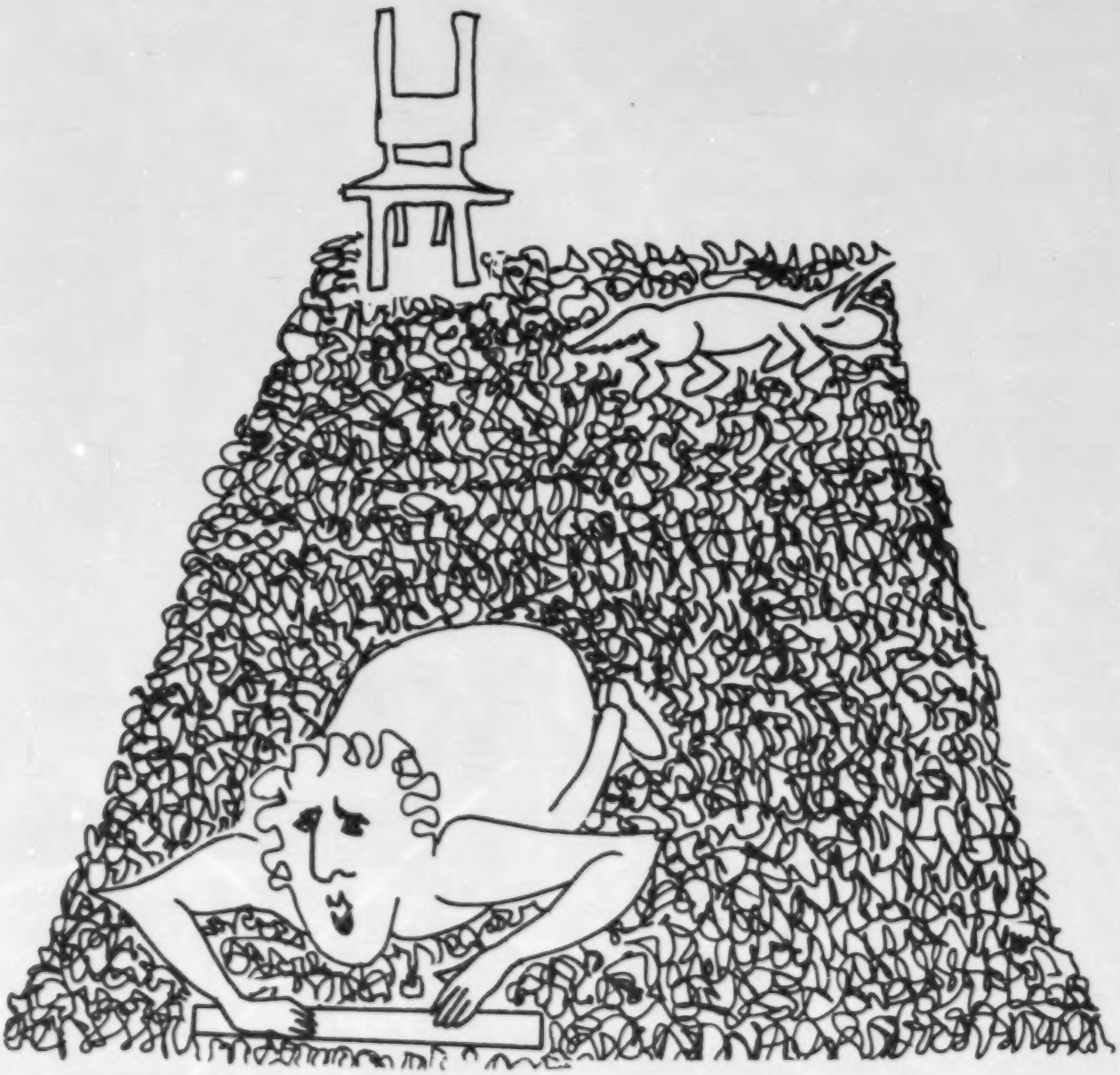
Janna Dory.

XVIII S-1 / XVIII S-2

Mrs. Brown needed to buy a new carpet for her living room. She took her yardstick and measured the old carpet. Here are its measurements:

length 16 feet

width 8 feet

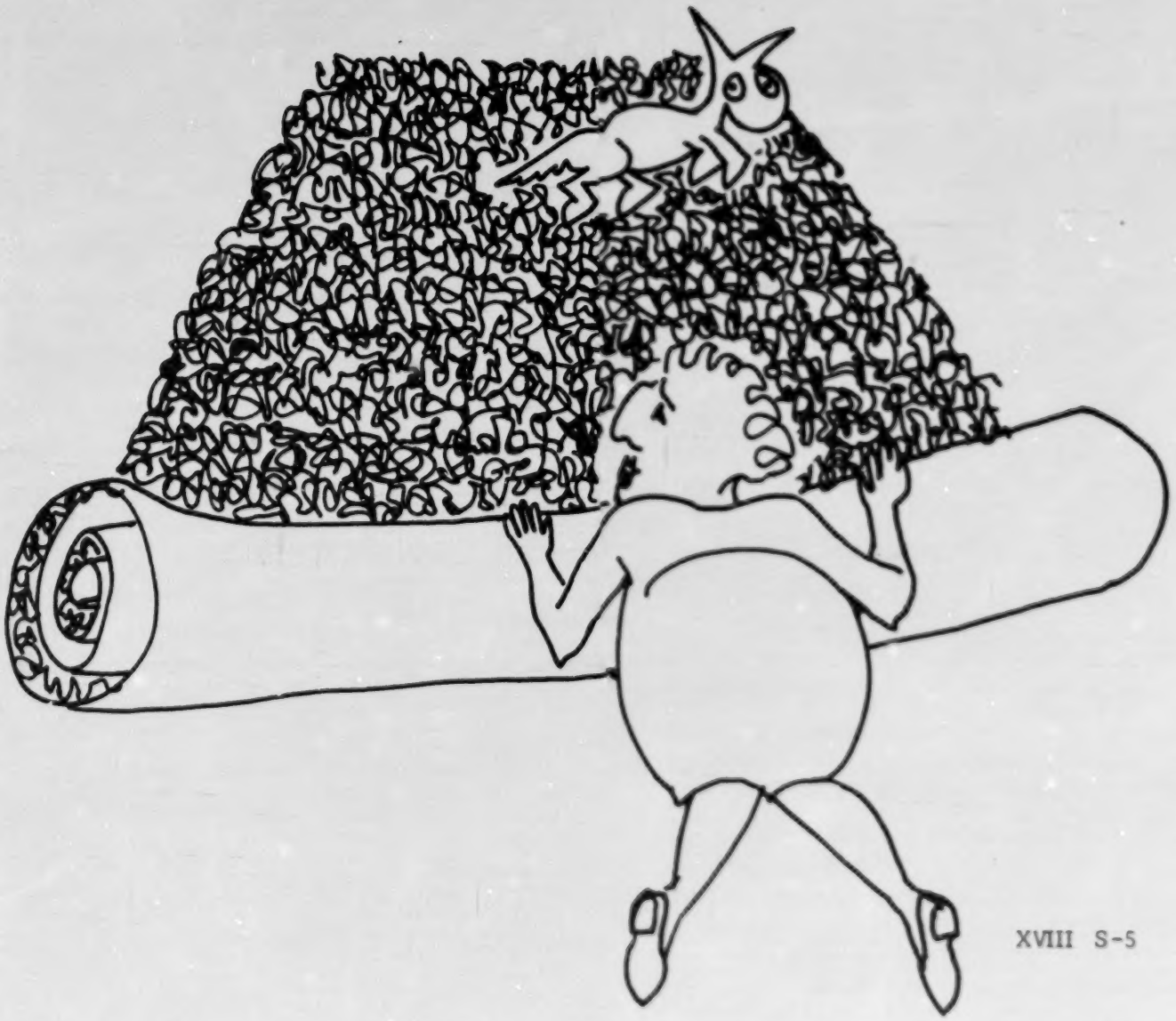


She took a piece of paper that was 8 1/2 inches wide and 11 inches long. "I shall make a scale drawing of this carpet," she said. "I'll let 1 inch on the scale drawing stand for 1 foot on my real carpet."

You can imagine how excited she was as she began her drawing!

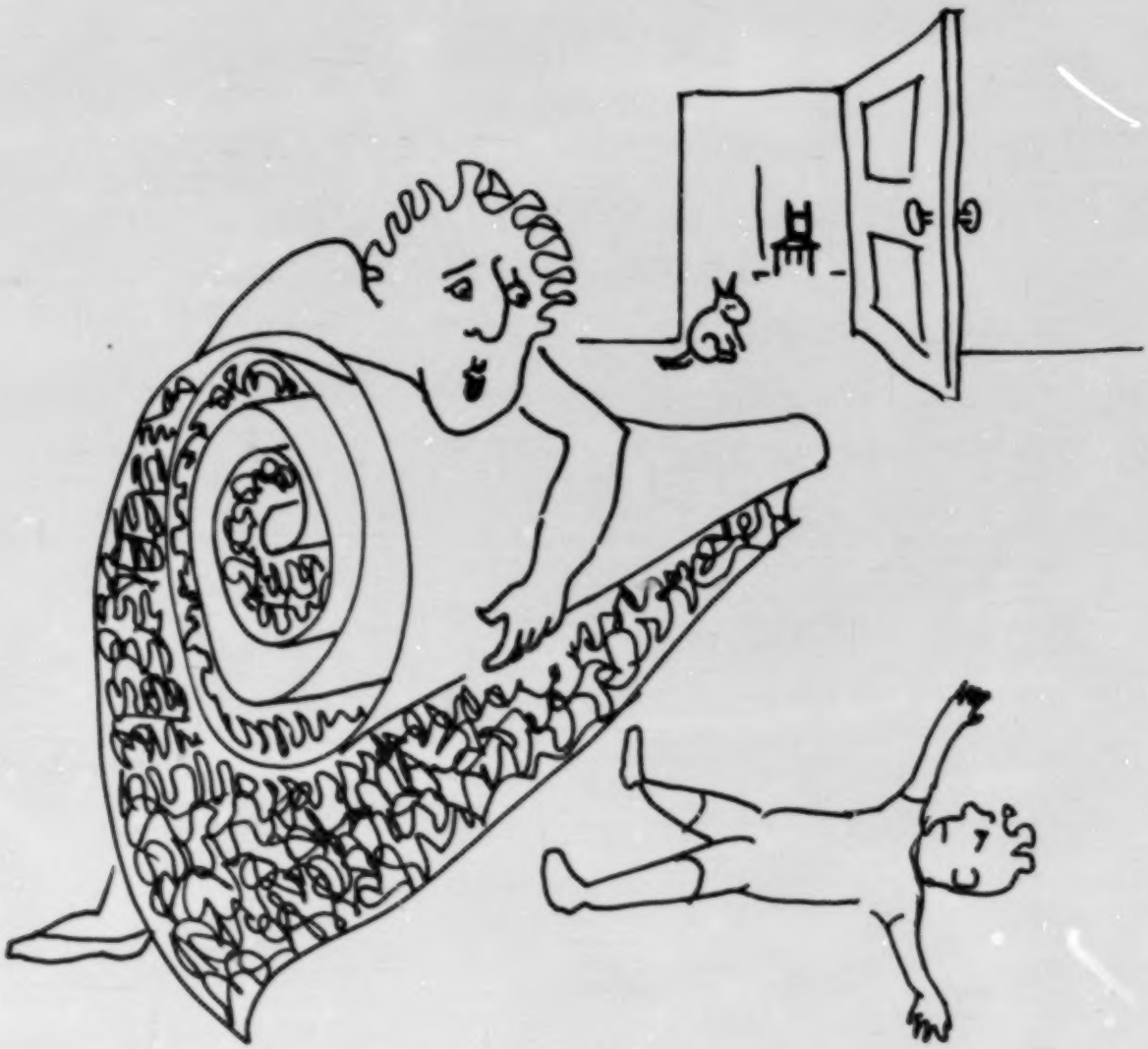
To make a long story short, she looked but couldn't find a large enough piece of paper on which to make her 1 inch$\leftrightarrow$1 foot scale drawing.

So, she rolled up her carpet, put it under her arm, heavy as it was and headed for the store.



As she went out the front door she met Tommy coming in. As you can imagine, a young boy is no match for a grown woman carrying a big carpet. BUMP, Tommy fell flat on the floor!

"This is very upsetting!" Tommy said as he picked himself up. "You shouldn't be carrying a big carpet like that! Just what do you intend to do with it?"

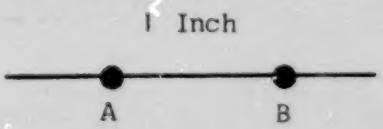


You know perfectly well what I'm planning to do," glared Tommy's Mother. "All of these pop and coffee spots are going to be replaced by a wonderfully soft, new carpet. Since you are personally responsible for some of these spots, it is your duty to help me carry this carpet to the store. So, grab ahold and let's go."

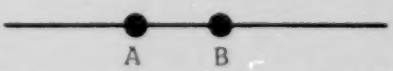
"Oh, no!" Tommy replied quickly. "This is too heavy to carry that far. Why don't you just make a scale drawing and take 'the scale drawing' to the store?"

"That, my fine young friend, has already been tried, "was the answer he received. "I just don't have a large enough piece of paper on which to make the scale unit smaller."

"Here, I'll show you," said Tommy. He picked up his notebook and took out a piece of paper. On it he made two marks that were about an inch apart.



He said, "It is about 1 inch from A to B. When you used this for your scale unit, the scale drawing did not fit on the paper. Try putting A and B closer together, like this:



Use the distance from A to B as your scale unit and see what happens."

Mrs. Brown used this new scale unit to make her new scale drawing. She was very surprised to find that her new scale drawing would fit on her sheet of paper. She smiled at Tommy and said, "Thank you, son. You'll be glad to know that you've just saved yourself a trip to the store."